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Applying nonlinear DEA models in order to increasing efficiency of electricity distribution companies

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Abstract. The aim of this study is designing a quantitative model to evaluate the efficiency of similar and homogeneous decision-making units through the development of a mathematical model called data envelopment analysis (DEA) so that it gives more precise figures for efficiency of decision making units than the current DEA models. According to the importance of the electricity industry and its importance, the subject of this thesis is the electricity distribution sector in Iran. In order to answer the main questions of this research, the methods of determining the multicollinearity of variables (using the index VIF (Variance Inflation Factor)) and Principle Component Analysis, linear and nonlinear estimates and expansion of the DEA model in a non-linear form model were used.

Key words. DEA models, Cobb-Douglas multiplicative model, efficiency, electricity distribution companies.

1. Introduction

Evaluating the productivity of DMU (Decision Making Unit), by measuring the ratio between the actual obtained outputs to the inputs is the first step in improving management. The importance of this issue is that, without measuring productivity, measure, and analyzes the status quo, taking the next steps is very difficult for reforming. After measuring productivity and identifying the current situation, we can decide to move from the current point to the desirable point.

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This research is looking for designing a quantitative model, to evaluate the efficiency of similar and homogeneous decision-making units, through the development of a mathematical model, called Data Envelopment Analysis (DEA). So that, compared to the current models, DEA gives more accurate productivity figures for similar decision-making units and provides better guidance on the recognition of the status quo.

Due to the effect of the electricity industry in the development of other sectors of the economy and the impact of its productivity on the performance of other sectors, case study of this thesis is related to one of the important segments of the electricity industry, meaning the electricity distribution sector. 39 electricity distribution companies active in Iran were considered as 39 homogeneous decision-making units, in the case study of this thesis which their rate of productivity should be calculated.

In this research, in addition to pursue the main objective that is the development of data envelopment analysis to non-linear form, other objectives are pursued, namely: a review of methods for assessing the productivity, a research on the relationship between inputs and outputs in electricity distribution companies and determine the best relationship, introducing the DEA and a review of studies in Iran and the world, using the basic model (linear) and expansion of proposed finding (non-linear), in evaluating the productivity of similar decision-making units, proving the existence of the global answer in the extended model and comparing the results of using conventional models and expanding DEA.

1.1. Productivity evaluation methods with emphasis on data envelopment analysis

Measuring productivity in a company or organization can be in the "simplest" form and in case of providing more information; it can be closed to sophisticated methods. There is no standard classification to classify the methods of measurement and different criteria are used for the classification of methods [1].

1. Some Organizations such as Organization Economic Cooperation and Development (OECD) divide output and input measures into two parts of physical and monetary and measure based on both.

2. Classification in terms of time and place. For example, C. Winston & Hall divided productivity measurement systems into two forms [2]:

    - The measurement of productivity in a period.
    - The relative measurement over a period of time.

3. Productivity divided into two groups of overall and partial marginal productivity is one of the divisions.

1.2. Expending DEA models applying for increasing efficiency

DEA model was introduced by Charnes, Cooper and Rhodes in 1978 [3]. This method is the generalized Farle method of two inputs and one output (1957) to systems with multiple input and multiple output and now it is used increasingly for evaluating the productivity of governmental and non-governmental organizations that contain a set of units or similar branches [4].

To obtain a formulation, which can be extended to multiple inputs and multiple outputs, we start as follows. Suppose, the DMU that is assessed, contains n units. Suppose that y_j is the output of the jth unit and x_j will be the input of the same unit, therefore, the DMU productivity of the pth unit is [5]

$$\frac{\frac{y_p}{x_p}}{\max_j \frac{y_j}{x_j}}, \ j = 1, 2, \cdots, n\,. \tag{1}$$

Its value cannot be more than one. The above formulation is input-based and the goal is reducing the input with maintaining the previous output to increase productivity [6].

The non-linear programming equation below

$$\max \frac{y_p}{x_p}\frac{u}{v}, \quad \max y_j x_j \frac{u}{v} \leq 1, \quad u, v \geq -, \quad j = 1, 2, \cdots, n\,. \tag{2}$$

has the same answer as (1). It is shown that for finding the answer of (3), the following equation can be solved:

$$\max z_p = y_p\, u, \quad x_p\, v = 1, \quad uy_j - vx_j \leq -, \quad u, v \geq -, \quad j = 1, 2, \cdots, n\,. \tag{3}$$

The above equation involves $n+1$ limits.

Suppose n Decision Making Units are available and each of them uses m different inputs to produce s different outputs. Suppose y_{rj}, x_{ij}, respectively, show the arrival of the ith $(i = 1, 2, \cdots, m)$ input and rth $(r = 1, 2, \cdots, s)$ output from the jth decision making unit $j = 1, 2, \cdots, n$ and also assume that every single decision making unit has at least one positive input and output (Meaning [7]), see Fig. 1.

Fig. 1. Input and output vectors of decision making unit

Here, x_j, y_j, respectively, are the jth input and output vectors of the unit. If we show corresponding weights of the output with μ_r, $r = 1, 2, ..., s$ and input with w_i, $i = 1, 2, ..., m$, in this case.

If we find the maximum of the above amount, then the equation will be unlimited, that is why we apply limitations to the equation and the productivity in all units.

By turning Charnes Cooper (1962), the equation of fractional planning will be converted into a linear programming problem [8]

$$\frac{1}{\sum w_i x_{ip}} = t\,,$$

$$t w_i = v_i,\ i = 1, 2, \cdots, m\,,$$

$$t\mu_r = u_r,\ r = 1, 2, \cdots, s\,. \tag{4}$$

In this case, we have

$$\max \sum_{r=1}^{s} u_r\, y_{rp}\,,$$

$$\text{sto} : \sum_{i=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0\,,$$

$$\sum_{i=1}^{m} v_i\, x_{ip} = 1\,.$$

$$u_r,\ v_i \geq \varepsilon,\ r = 1, 2, 3, \cdots, s,\ i = 1, 2, 3, \cdots, m\,. \tag{5}$$

In fact, θ shows the reduction maximum in line with input, because this reduction ratio is the same in all input directions. Therefore, it shows positive covariance representing more reduction in some directions. In this case, θ represents the minimum level of inputs to achieve a suitable output level. That is why we call θ as the productivity of the unit under assessment. There holds

$$\theta* = 1,\ \forall i,\ \forall r\ sr^{+} = si^{-} = 0\,. \tag{6}$$

Problem (5) or (6) must be solved on the number of decision-making units so that productive and nonproductive units and productivity sources will be determined. In the written model that turns a non-productive unit into a productive unit is as follows [10]:

$$(x_p, y_p) \rightarrow (\theta * x_p - s^{-}*, y_p + s^{+}*)\,. \tag{7}$$

If we assume that all studied n units will be under the supervision of a centralized management and form a complex, then a studied unit of the n units, will be non-productive, if the output of the unit $>$ Complex output and the consumed input of the unit $<$ Complex input.

This means that the management of the unit could not use the opportunities available to it, one hundred percent.

According to the model (7), the DEA method, with assuming the creation of a hypothetical center, made a linear programming model that the output of the center is output-weighted combination of similar units [11]. Because of this method, the goal is to find non-productive unit, therefore, we show that the weighted output of the collection is more than the considered unit output [12]. However, since the unit has not worked with high productivity, the model shows us that the relative productivity of the unit is less than one [13].

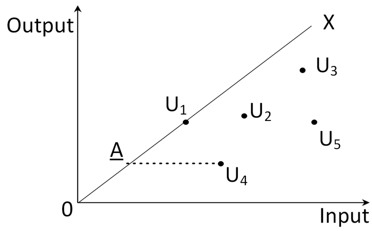

Fig. 2. Basic input model

As it is evident in Fig. 2, decision-making unit U_1, because it is placed on the productivity frontier (ox), is productive and other units are non-productive. In fact, in model (7), it will be tried to reduce input (without reducing the output) in order to identify how a non-productive unit becomes productive [14]. The interpretation of this in Fig. 2 means the placement of the decision-making unit U_4 to point A on the border of productivity [15].

If the aim will be increasing output for every DMU, then the CCR model will be obtained with output nature, that its modeling for evaluating DMU_p as follows:

$$\min \sum_{i=1}^{m} v_i\, x_{ip}\,,$$

$$\text{sto}: \sum_{r=1}^{s} u_r\, y_{rp} = 1\,,$$

$$\sum v_i\, x_{ij} - \sum u_r\, y_{rj} \geq -, \quad j = 1, 2, \cdots, n\,,$$

$$u_r,\ v_i \geq \varepsilon, \quad r = 1, 2, \cdots, s, \quad i = 1, 2, \cdots, m\,. \tag{8}$$

The dual problem to the above problem is as follows:

$$\max \emptyset + \varepsilon \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right)\,,$$

$$\text{sto}: \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip},\ i = 1, 2, \cdots, m\,,$$

$$\sum \lambda_j y_{rj} - s_r^+ = \emptyset y_{rp}, \quad r = 1, 2, \cdots, s\,,$$

$$\lambda_j \geq -,\ j = 1, 2, \cdots, n\,,$$

$$s_i^- \geq - i = 1, 2, \cdots, m\,,$$

$$s_r^+ \geq -r = 1, 2, \cdots, s\,. \tag{9}$$

In fact, $\emptyset$ by multiplying y_{rp} tries to increase it, so that the input does not increase. We call $\emptyset$ the productivity amount of the pth unit and we can prove that $\emptyset * = 1/\theta *$, where * represents the optimum. To image it for the previous example, means moving decision-making unit U_4 to point B on the frontier of productivity that is shown in Fig. 3.

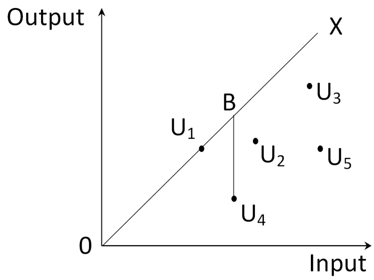

Fig. 3. Basic output model

In fact, if DMU_p will be non-productive, then it can be productive with the below change in the output nature [15]

$$(x_p, y_p) \to (x_p - s^-,\, \emptyset_p y_p + s^+)\,.$$

1.3. The method for selecting independent variables on linear and nonlinear DEA proposed model

Modeling to determine the relationship between the variables in case of depending of independent variables to each other can be misleading. Understanding of the relationships' impact between variables is important in the analysis.

1.3.1. Linearity. Two variables x_1 and x_2 are linear, when there exits constants c_1, c_2 and c such that

$$c_1 x_1 + c_2 x_2 = c\,, \tag{10}$$

which is correct for all kinds of states of data [16]. Imperfect multicollinearity is achieved when this equation is established approximately for the observed data. The sample correlation square, which means r_{12}^2 is a common degree, but not completely appropriate for the multicollinearity degree between x_1 and x_2. Perfect multicollinearity relates to $r_{12}^2 = 1$ and nonlinearity relates to $r_{12}^2 = 0$. We generalize

$$c_1 x_1 + c_2 x_2 + \cdots + c_p x_p = c\,. \tag{11}$$

A simple diagnostic method is the square of multiple correlations between x_k and other x_j that is called R_{kj}^2 and this number is calculated from x_k regression with respect to other x_j. If the highest R^2_{kj} will be one, the diagnosis of multicollinearity is incomplete.

It is observed that coefficient in nonlinear models is more than the linear model, whether they are considered all independent variables or for reduced models. Among non-linear models, respectively, multiplicative models of Cobb–Douglas, grade 2 and multiplicative exponential have a greater coefficient of determination [17].

It is noteworthy that the coefficient of determination (R^2) is appropriate for comparing models, when the error terms follow a normal distribution criterion. If these words do not follow the normal distribution, these criteria cannot properly maintain its credibility [18]. For studying the normal distribution of the error, the cumulative probability qq-plot will be used that draws the remaining values in corresponding percentiles with any remains in normal distribution. If no big deviation from line $y = x$ will be observed, we can trust the normal distribution of the error terms, of course, when this deviation is negligible in applications to a proper limit [19].

1.4. Suggestion of extending the DEA model to non-linear form of the Cobb–Douglas multiplicative type

In non-parametric methods, including data envelopment analysis, calculation the productivity of similar decision-making units is by determining the technical relationships between inputs and outputs rather than estimating the production function. In these methods, a border is considered for decision-making units, according to the values of inputs and outputs as criteria for productivity [20]. In the DEA method, we obtain weight for each of the outputs (products) that the weight shows the produced output desirability. Then by multiplying each weight with the considered output value, we calculate the total of these multiplications that are used as a weighted combination of outputs [21].

This total number, in principle, is a linear function of the outputs (product). Similarly, we obtain a weight for each of the inputs and by multiplying each weight by corresponding input value; we calculate the total multiplications that are used as a weighted combination of inputs [22]. This total number is essentially a linear function of the inputs. By calculating the ratio of weighted outputs to the weighted inputs, the weighted rate of productivity will be identified. According to the results in the third chapter and preferred non-linear relationships to linear relations for DEA model, nonlinear model of Cobb-Douglas multiplicative type is recommended in the form

$$\min w_p = V \prod_{i=1}^{m} x_{ip}^{vi} \prod_{r=1}^{s} y_{rp}^{-u_r}$$

$$\text{sto} : v^{-1} \prod_{i-1}^{m} x_{ij}{}^{-vi} \prod_{r=1}^{s} y_{rj}^{+u_r} \leq 1, \quad u, v \geq 1^{\times}, \quad j = 1, \cdots, n\,. \tag{12}$$

In the above equation, w_p is the productivity of the pth decision-making unit, n is the number of DMUs, s is the number of outputs, m is the number of inputs, u_r is the weight (importance) of the rth output, v_i is the weight (importance) of

ith input, p stands for the index of the pth DMU, j represents the index of the jth DMU, n is the number of limitations and $1^{\times}$ denotes the vector containing the units. The number of variables is $m + s + 1$ [23].

1.5. Representing and solving an example through a non-linear DEA model (primary and dual)

Example: In order to compare the productivity of 4 decision-making units that each one has two inputs and two outputs, using the linear-based and output-based DEA model (CCRp -O) and the expanded model of Cobb–Douglas, the following example will be:

$$\min Z = \sum_{i=1}^{2} v_i \, x_{ip} \,,$$

$$\text{sto} : \sum_{r=1}^{2} u_r \, y_{rp} = 1 \,,$$

$$\sum_{i=1}^{2} v_i \, x_{ij} - \sum_{r=1}^{2} u_r \, y_{rj} \geq 0, \quad j = 1, 2, 3, 4 \,,$$

$$v_i, u_r \geq \varepsilon, \quad r, i = 1, 2, \quad \varepsilon = 10^{-6} \,. \tag{13}$$

This solution results from the model for four decision-making units that have been solved with WINQSB software are written in Table 1 [24]. Since, an output-based model has been used, we expect that Z for productive units will be equal to 1 and for non-productive units will be greater than one. As it is shown in Table 1, decision-making units No. 1 and 3 are productive and decision making units No. 2 and 4 are non-productive.

Table 1. Example, the input and output amounts

DMUp	The first input x_{1p}	The second input x_{2p}	The first output y_{1p}	The second output y_{2p}
DMU_1	4	1.5	10	12
DMU_2	3	3	8	9
DMU_3	1	4	9	6
DMU_4	5	4	7	8

2. Designing Cobb-Douglas multiplicative DEA model

In this research, that was done with the aim to design a quantitative model for evaluating the productivity of similar and homogeneous decision-making units through the mathematical model of data envelopment analysis development. A review of methods for evaluating productivity has been done and due to the bugs made with traditional methods, data envelopment analysis came into focus [25]. Data envelopment analysis is a nonparametric method in order to estimate the production function. After studying the course and the emergence of DEA development in the fields of theoretical, applied in Iran and the world, the input, and output factors in electricity distribution companies have been investigated in previous studies. Independence inputs of Electricity distribution companies were tested through variance inflation factor (VIF) and it was found that input factors do not have multi-collinearity and in other words, they are independent of each other [26].

In order to determine more suitable and more expressive relationships between inputs and outputs of electricity distribution companies in Iran, the regression analysis was used and it was found that nonlinear relations, especially relations of multiplicative Cobb–Douglas would express the inputs and outputs behavior of the electricity distribution companies in Iran better than linear relationship [27]. According to these analyses and other primers in this research, extended suggestion of the DEA model has been presented in non-linear form and with Multiplicative Cobb–Douglas type.

2.1. The theoretical basis

This approach is because data envelopment analysis is a non-parametric method based on mathematical programming to estimate the production function of similar decision-making units [28]. On the other hand, in the third chapter, through regression analysis, different production functions were examined in a case study that the production function of Cobb–Douglas presented a better response than other forms of production function.

2.2. Necessity for global answer (across) in the non-linear DEA model

In order, a practical area creates a convex set for nonlinear programming; the convex restrictions should be specified in the problem with lower or equal relation ($\leq$). For example, if the limitation function, $g_i(x)$ is for a convex nonlinear problem and the limitation is specified ($b_i \geq 0$), $g_i\left(x\right) \leq b_i$, for two practical points of x_1, x_2, with respect to the definition of convex function of it, we have:

$$g_i\left[\lambda x_1 + (1+\lambda)\,x_2\right] \leq \lambda g_i\left(x\right) + (1-\lambda)\,g_i\left(x\right) \leq \lambda b_i + (1+\lambda)\,b_i = b_i\,. \tag{14}$$

As a result, if x_1 and x_2 will be practical, then any linear combination of these points is also practical. Therefore, the constraints $g_i(x) \leq b_i$ forms a convex and practical set. According to the case, in a nonlinear program consisted of a convex objective function (to minimum) and a convex practical area that is called convex programming, the local minimum point will be the global minimum point [29].

In the proposed model of this research, statements in the objective function are posynominal. Dauphine, Clarence and Vezner showed that the simple variable change shows each posynominal as a convex function.

$$f\left((1-\theta)\,x+\theta y\right)\leq(1-\theta)\,f\left(x\right)+\theta f\left(y\right)\,, \tag{15}$$

where θ is a number between one and zero.

This is equivalent to the following statements:

$$g\left(\theta\right)\leq(1-\theta)\,g\left(\theta\right)+\theta y\left(1\right),\quad 0\leq\theta\leq 1\,. \tag{16}$$

The above inequality is equivalent to 15 and means that the function $f(z)$ is convex, when

$$\frac{\mathrm{d}^2 g}{\mathrm{d}\theta^2}=g''\left(\theta\right)\geq 0\,.$$

Therefore, $f(z)$ will be convex if

$$\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\right)^2 f((1-\theta)\,x+\theta y)\geq 0,\quad 0\leq\theta\leq 1\,. \tag{17}$$

This will provide the following result:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}u_i\left(\partial_{ij}f\right)u_j\geq 0\,. \tag{18}$$

where $u_i=y_i-x_i$ and ∂_{ij} is the second derivative of $f(x)$ with respect to z_j and z_i. If H is a symmetric matrix with elements $\partial)ij$, in this case, this inequality can be expressed as

$$U\cdot H\cdot U\geq 0\,. \tag{19}$$

In the case of this condition, semi-definite matrix of second order derivatives of f is positively definite and f function is convex. Posynominal sentences in geometric programming usually are as follows:

$$p(t)=\sum_{*=1}^{m}c\prod_{j=1}^{n}t_j{}^{kj}\,. \tag{20}$$

It means the sentence * of the function $p(t)$ is posynominal and it is shown as

$$p*\left(t\right)=C*\prod_{j=1}^{n}t_j{}^{kj}\,. \tag{21}$$

With a simple change, we can prove that $p*(t)$ is convex. Since in the posynominal $c*>0$ and t_j >0, k_j should be a real number, so that the following change in the variable can be done:

$$t_1=\mathrm{e}^{x_1},\,t_2=\mathrm{e}^{x_2},\,\cdots..,\,t_n=\mathrm{e}^{x_n}\,. \tag{22}$$

Therefore

$$f(x) = p*(t) = C*\mathrm{e}^{k_1x_1}\mathrm{e}^{k_2x_2}\cdots\mathrm{e}^{k_nx_n} =$$

$$= C*\mathrm{e}^{(k_1x_1+k_1k_2+\cdots+k_nx_n)} = C*\mathrm{e}^{kx}\,. \tag{23}$$

On the other hand

$$\partial_{ijf} = C*k_ik_j\mathrm{e}^{kx} = k_ik_jf(x)\,. \tag{24}$$

Therefore

$$\sum_i\sum_j u_i(\partial_{ijf})u_j = \sum_i\sum_j u_i(k_ik_jf(x))u_j = \sum_i u_ik_j\sum_j u_jk_jf(x) =$$

$$= \left(\sum_s u_sk_s\right)^2 f(x) = \left(\sum_s u_sk_s\right)^2 \left(C*\mathrm{e}^{kx}\right)\,. \tag{25}$$

Since $(\sum u_sk_s)^2\left(C*\mathrm{e}^{kx}\right)$ is always greater than or equal to zero ($C* > 0$), then $\sum_i\sum_j u_i\left(\partial_{ij}f\right)u_j$ will always be greater than or equal to zero. Therefore, $f(x)$ will be convex.

3. Conclusion

The aim of this study is designing a quantitative model to evaluate the efficiency of similar and homogeneous decision-making units through the development of a mathematical model called data envelopment analysis (DEA) so that it gives more precise figures for efficiency of decision making units than the current DEA models.

A study of linear and nonlinear estimates on data collection of the study shows that the nonlinear relations, especially relations of Cobb-Douglas multiplicative shows inputs and outputs behavior to each other better than to linear relations.

According to the results, the offer to extend from the DEA model in non-linear and Cobb-Douglas multiplicative type has been presented and it has been shown that the extended model has global solution and its solving method, which is geometric planning, has been introduced.

It is shown that the proposed model has a global optimum answer (throughout) and with the methods of Geometric programming as an appropriate and consistent method to the proposed model.

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